# Mean Free Path of Emitted Molecules and Correlation of Sphere Drag Data

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**The mean free path between molecules emitted from and incident to a surface in rarefied flow is considered. The problem is formulated for the case of two classes of hard sphere molecules with different Maxwellian velocity distribution functions. Approximate analytical solutions are obtained to the integral describing the collision frequency between emitted and incident molecules. These analytical solutions are compared with numerical solutions, and the domain of validity of the analytical results is established. The analytical solutions provide a simple expression for the mean free path which is used to define a correlation parameter. This parameter is used to correlate a great deal of low-density supersonic and hypersonic sphere drag data which have a wide range of freestream and body surface conditions.**

## Nomenclature

$A$ = cross-sectional area
$C_D$ = drag coefficient, $D/(q_\infty A)$
$c_p$ = specific heat of gas at constant pressure
$c_v$ = specific heat of gas at constant volume
$D$ = drag force
$d$ = body diameter
$\mathrm{erf}(x)$ = error function, $[2/(\pi)^{1/2}]\int_0^x \exp(-t^2)\,dt$
$f$ = velocity distribution function
$g$ = relative speed of molecules
$h$ = defined by Eq. (5)
$I$ = defined by Eq. (9)
$Kn$ = Knudsen number
$\bar{K}$ = correlation parameter, $Kn_\infty/[(\pi)^{1/2}S_w+1]$
$M$ = Mach number
$m$ = mass of a molecule
$N_{12}$ = collision frequency per unit volume between class 1 and 2 molecules
$n$ = number density of molecules
$\mathbf{P}$ = vector defined by Eq. (6)
$p$ = pressure
$\mathbf{Q}$ = vector, $\mathbf{Q} = (\omega_1, -\omega_2, 0, 0, 0, 0)$
$q$ = dynamic pressure, $\rho U^2/2$
$R$ = gas constant
$Re$ = Reynolds number
$r$ = body radius
$S$ = speed ratio, $U/(2RT)^{1/2}$
$S_r$ = $U_\infty/(2RT_r)^{1/2}$
$S_w$ = $U_\infty/(2RT_w)^{1/2}$
$S_\infty$ = $U_\infty/(2RT_\infty)^{1/2}$
$T$ = temperature
$T_r$ = reference temperature, $(T_o^- + T^+)/2$
$U$ = velocity
$u$ = molecular velocity component
$v$ = molecular velocity component
$\bar{v}$ = mean speed of molecules
$w$ = molecular velocity component
$\beta$ = angle between the velocity vectors $U$ and $v$
$\gamma$ = ratio of specific heats, $c_p/c_v$
$\zeta$ = $w/(2RT)^{1/2}$
$\eta$ = $v/(2RT)^{1/2}$
$\lambda$ = mean free path
$\mu$ = viscosity
$\xi$ = $u/(2RT)^{1/2}$
$\rho$ = gas mass density
$\sigma_{12}$ = $(\sigma_1+\sigma_2)/2$
$\sigma$ = molecular diameter
$\tau$ = $(T_2/T_1)^{1/2}$ and $(T_w/T_\infty)^{1/2}$
$\omega_1$ = $S_1 \sin\beta$
$\omega_2$ = $S_1 \cos\beta$

*Subscripts*

1 = class 1 molecules
2 = class 2 molecules
$d$ = body diameter
$fm$ = free-molecular value
$i$ = can be either 1 or 2
$o$ = reservoir (total) conditions
$w$ = body wall condition
$\infty$ = freestream condition

*Superscripts*

$-$ = incoming molecules
$+$ = outgoing molecules
(0) = zeroth order
(1) = first order
(2) = second order

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## I. Introduction

THE mean free path of molecules emitted from the surface of a body before experiencing collisions with molecules incident to the body, $\lambda_w$, was shown to be important in several theoretical and experimental investigations of rarefied flow problems.[1–8] For example, the significance of this mean free path is clearly pointed out by Probstein.[2] Explicit calculations of $\lambda_w$ have been based on the assumption that all molecules are emitted from the surface at a constant mean speed and, in some cases, restricted to cold surfaces. It is of interest, therefore, to consider more fundamentally the deviation of $\lambda_w$ where the relative velocity between colliding molecules is not constant and the surface is not necessarily cold.

The approach taken is to formulate the problem for two classes of hard sphere molecules with different Maxwellian velocity distribution functions. Approximate analytical solutions to the integral describing the collision frequency between the two classes of molecules are obtained and compared to numerical solutions. The domain of validity of the approximate results is established and the accuracy assessed. The analytical results are also compared to the result corresponding to using a mean relative velocity between colliding molecules. It is found that the present first-order solution for $\lambda_w$ is equally as simple as the expression based on a mean relative velocity, and it is more accurate.

The analytical results provide a convenient expression for $\lambda_w$ which is used to establish a correlation parameter. This parameter is used to correlate a large number of low-density supersonic and hypersonic sphere drag data. A critical test of the role of wall temperature in the parameter is provided by supersonic drag data[9] taken in constant flow conditions, and only the sphere wall temperature was changed. This parameter is also applied to the correlation of high-enthalpy, hypersonic, cold-wall sphere drag data. It was recently pointed out[9,10] that such data have larger drag coefficients than what might be expected, based on lower Mach number and/or lower-enthalpy drag data. The use of an effective or reference temperature for emitted molecules has been discussed by various investigators[5,6,11,12] in relating certain parameters to physical variables. It is found in the present investigation that the reference temperature, $T_r$, derived by Kinslow and Potter[13] and used to correlate surface pressures in high-enthalpy hypersonic flows, provides a means of correlating sphere drag data in similar flows if $T_w$ is replaced by $T_r$ in the present parameter. In fact, it is found that high and low enthalpy, supersonic and hypersonic, and hot and cold wall sphere drag data can be consistently correlated by using the reference temperature of Kinslow and Potter.[13]

## II. Analysis

### Formulation

Consider two classes of smooth, rigid, elastic, spherical molecules denoted as 1 and 2. For such molecules, no problem exists concerning the concept of a free path since the molecules affect each other's motion only at collisions. Following the development of Chapman and Cowling,[14] the total number of collisions occurring per unit volume and unit time between pairs of molecules of class 1 with class 2 is given by

$$N_{12} = \int\int\int\int\int\int \pi g \sigma_{12}{}^2 f_1 f_2 \, du_1 \, dv_1 \, dw_1 \, du_2 \, dv_2 \, dw_2 \tag{1}$$

The Maxwellian distribution functions for these two classes of molecules are

$$f_1 = \frac{n_1}{(2\pi R T_1)^{3/2}} \exp\left\{ -\frac{1}{2RT_1}\left[(u_1 - U\sin\beta)^2 + (v_1 + U\cos\beta)^2 + w_1{}^2\right]\right\} \tag{2}$$

and

$$f_2 = \frac{n_2}{(2\pi R T_2)^{3/2}} \exp\left[ -\frac{1}{2RT_2}(u_2{}^2 + v_2{}^2 + w_2{}^2)\right] \tag{3}$$

The coordinate system is fixed with respect to a surface element where the $u_i$ velocity components are directed inward and normal to the surface, $v_i$ are tangent to the surface and lie in the plane of the mean velocity $U$ and the components $u_i$, $w_i$ are normal to $u_i$ and $v_i$, and $\beta$ is the angle between $U$ and $v_i$. Nondimensionalizing by the variables as defined in the nomenclature, Eq. (1) can be written as

$$N_{12} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{2\pi\sigma_{12}{}^2 n_1 n_2 (2RT_1)^{1/2}}{\pi^3} \times h(\mathbf{P}) \exp\{-[(\xi_1-\omega_1)^2 + (\eta_1+\omega_2)^2 + \zeta_1{}^2 + \xi_2{}^2 + \eta_2{}^2 + \zeta_2{}^2]\} \times d\xi_1\, d\eta_1\, d\zeta_1\, d\xi_2\, d\eta_2\, d\zeta_2 \tag{4}$$

where

$$h(\mathbf{P}) = [(\xi_1 - \tau\xi_2)^2 + (\eta_1 - \tau\eta_2)^2 + (\zeta_1 - \tau\zeta_2)^2]^{1/2} \tag{5}$$

and

$$\mathbf{P} = (\xi_1, \eta_1, \zeta_1, \xi_2, \eta_2, \zeta_2) \tag{6}$$

The limits of integration of Eq. (4) for $\xi_1$ and $\xi_2$ correspond to considering incident (class 1) and emitted (class 2) molecules. One would note by following the details of the transformation, an additional 2 as a coefficient in the integrand of Eq. (4). This 2 is required to satisfy continuity since the limits of $\xi_2$ are $-\infty$ to 0, and hence only one-half of the molecules emitted from the surface element would otherwise be accounted for. The mean distance traveled by molecules of class 2 between successive collisions with those of class 1 is given by[14]

$$\lambda_{21} = n_2 \bar{v}_2 / N_{12} \tag{7}$$

where $\bar{v}_2$ is the mean speed of class 2 molecules. Taking $\bar{v}_2 = (9\pi R T_2/8)^{1/2}$, (Ref. 3), $\lambda_{11} = [(2)^{1/2}\pi n_1 \sigma_{11}{}^2]^{-1}$, (Ref. 14), and $\sigma_1 = \sigma_2$, one obtains

$$\lambda_{11}/\lambda_{21} = [8/3\pi^3(2\pi)^{1/2}](I/\tau) \tag{8}$$

where

$$I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} h(\mathbf{P}) \exp\{-[(\xi_1-\omega_1)^2 + (\eta_1+\omega_2)^2 + \zeta_1{}^2 + \xi_2{}^2 + \eta_2{}^2 + \zeta_2{}^2]\}\, d\xi_1\, d\eta_1\, d\zeta_1\, d\xi_2\, d\eta_2\, d\zeta_2 \tag{9}$$

and $h(\mathbf{P})$ is given by Eq. (5). The solution for the mean free path requires the integration of Eq. (9), which is proportional to the collision frequency between class 1 and class 2 molecules.

### Approximate Analytical Solutions

Consider the integrand of Eq. (9). It is non-negative, and due to the exponential term it attains its larger values at relatively small values of the independent variables $\zeta_1$, $\xi_2$, $\eta_2$, and $\zeta_2$, and values of $\xi_1$ and $\eta_1$ near $\omega_1$ and $-\omega_2$, respectively. Therefore, if an acceptable approximation to the integrand of Eq. (9) can be obtained, particularly in the neighborhood of where the integrand is largest, and if this approximating function can be integrated, then an expression for $I$ can be obtained. This was the motivation for the present approach.

Expanding $h(\mathbf{P})$, Eq. (5), in a Taylor series in its six independent variables about the point $\mathbf{Q} = (\omega_1, -\omega_2, 0, 0, 0, 0)$, one obtains for the zeroth-, first-, and second-order approximations of $h(\mathbf{P})$ the expressions

$$h^{(0)}(\mathbf{P}) = h(\mathbf{Q}) = (\omega_1{}^2 + \omega_2{}^2)^{1/2} = S_1 \tag{10}$$

$$h^{(1)}(\mathbf{P}) = h^{(0)}(\mathbf{P}) + (\omega_1/S_1)(\xi_1 - \omega_1 - \tau\xi_2) - (\omega_2/S_1)(\eta_1 + \omega_2 + \tau\eta_2) \tag{11}$$

$$h^{(2)}(\mathbf{P}) = h^{(1)}(\mathbf{P}) + \frac{1}{2S_1}\left\{\left[\frac{\omega_1}{S_1}(\eta_1 + \omega_2 - \tau\eta_2) + \frac{\omega_2}{S_1}(\xi_1 - \omega_1 - \tau\xi_2)\right]^2 + (\zeta_1{}^2 + \tau^2\zeta_2{}^2)\right\} \tag{12}$$

Using these approximations in Eq. (9), the zeroth-, first-, and second-order results for $I$ can be obtained (see, for example, Patterson[15] for the evaluation of such integrals). Using the appropriate expression for $I$ in Eq. (8), one obtains the following results for the zeroth-, first-, and second-order ratios of mean free paths

$$\left(\frac{\lambda_{11}}{\lambda_{21}}\right)^{(0)} = \frac{(2\pi)^{1/2} S_2}{3\pi}[1 + \mathrm{erf}(\omega_1)] \tag{13}$$

$$\left(\frac{\lambda_{11}}{\lambda_{21}}\right)^{(1)} = \frac{(2)^{1/2}}{3\pi}\left\{[1 + \mathrm{erf}(\omega_1)]\left((\pi)^{1/2} S_2 + \frac{\omega_1}{S_1}\right) + \frac{\omega_1}{S_1 \tau}\exp(-\omega_1{}^2)\right\} \tag{14}$$

$$\left(\frac{\lambda_{11}}{\lambda_{21}}\right)^{(2)} = \frac{(2)^{1/2}}{3\pi}\left\{[1 + \mathrm{erf}(\omega_1)]\left[(\pi)^{1/2} S_2 + \frac{\omega_1}{S_1} + \frac{(\pi)^{1/2}}{2S_2}\left(1 + \frac{1}{\tau^2}\right)\right] + \frac{1}{2\tau}\exp(-\omega_1{}^2)\left[\left(\frac{\omega_2}{S_1}\right)^2 \frac{2}{(\pi)^{1/2} S_2} + \frac{\omega_1}{S_1} + \left(\frac{\omega_1}{S_1}\right)^3\right]\right\} \tag{15}$$

where $S_2 = S_1/\tau$.

## III. Results and Discussion

### Comparison of Analytical and Numerical Solutions

Assuming the molecules emitted from the surface correspond to the surface temperature, $T_w$, then perhaps a more familiar nomenclature would be $\lambda_{21} = \lambda_w$, $\lambda_{11} = \lambda_\infty$, $\tau = (T_w/T_\infty)^{1/2}$, $S_1 = S_\infty$, and $S_2 = S_w = S_\infty/\tau$. Numerical solutions of Eq. (9) have been presented[9] for the special case of $\beta = \pi/2$. Molecules emitted near $\beta = \pi/2$ are particularly significant in rarefied flow

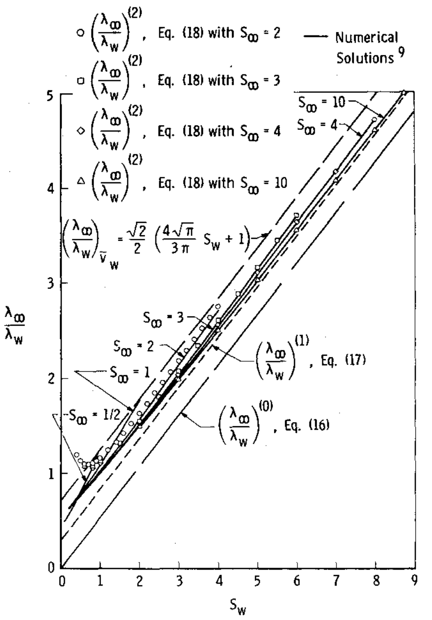


Fig. 1 Numerical and approximate analytical solutions for $\lambda_\infty/\lambda_w$ with $\beta = \pi/2$.

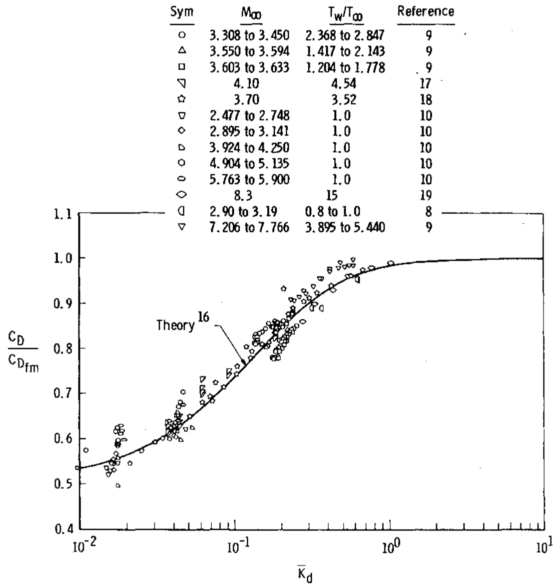


Fig. 2 Correlation of supersonic sphere drag data.

drag problems[3,7,16] due to their "shielding" effect. Also, it was shown in Ref. 16 that $\lambda_w/\lambda_\infty$ is relatively constant for $S_w > 1$ over the domain $\pi/6 < \beta < \pi/2$. The restriction $S_w > 1$ is not stringent. In fact, because $T_w$ is nearly always $T_\infty \leqq T_w \leqq T_o$ for supersonic wind tunnel or flight data, one has $S_w > 1$; or, for $T_w = T_o$ and relatively large $S_\infty$, one has $S_w > [\gamma/(\gamma-1)]^{1/2}$. For supersonic flow and $\beta = \pi/2$, Eqs. (13–15) are approximated by

$$\left(\frac{\lambda_\infty}{\lambda_w}\right)^{(0)} = \frac{(2)^{3/2}}{3\pi}[(\pi)^{1/2}S_w] \tag{16}$$

$$\left(\frac{\lambda_\infty}{\lambda_w}\right)^{(1)} = \frac{(2)^{3/2}}{3\pi}[(\pi)^{1/2}S_w+1] \tag{17}$$

$$\left(\frac{\lambda_\infty}{\lambda_w}\right)^{(2)} = \frac{(2)^{3/2}}{3\pi}\left[(\pi)^{1/2}S_w+1+\frac{(\pi)^{1/2}}{2S_w}\left(1+\frac{S_w^2}{S_\infty^2}\right)\right] \tag{18}$$

Equations (16–18) are compared with the numerical results[9] in Fig. 1. Reasonable agreement between Eq. (18) and the numerical results is obtained for $S_w > 1$. Also included in Fig. 1 is the result, denoted as $(\lambda_\infty/\lambda_w)_{\bar{v}_w}$, corresponding to a constant mean relative velocity for $g$ in Eq. (1). This expression has the same slope as Eqs. (16) and (17), but it gives values of $\lambda_\infty/\lambda_w$ above the present results and, in contrast to Eq. (17), does not have the proper limit for large $S_\infty$.

Although Eq. (18) is in better agreement with the numerical results than the other equations plotted in Fig. 1, it is not as convenient to use as, for example, Eq. (17). Furthermore, Eq. (17) eliminates the use of the additional parameter $S_\infty$. In view of its simple form and the fact that it is in fair agreement with the numerical results, particularly for the larger values of $S_\infty$ and for $S_w \geqq [\gamma/(\gamma-1)]^{1/2} \sim 0(1)$ which are of interest here, Eq. (17) will be used to establish a correlation parameter and investigate its applicability for correlating experimental drag data.

### Correlation Parameter

As pointed out, several investigations of low-density drag problems have shown that $\lambda_w$, in conjunction with a characteristic dimension, is an important parameter. For spheres the radius is appropriate and using Eq. (17) one can write

$$\frac{\lambda_w}{r} = \frac{3\pi}{(2)^{3/2}}\frac{\lambda_\infty/r}{(\pi)^{1/2}S_w+1} = \frac{3\pi}{(2)^{1/2}}\frac{Kn_{\infty,d}}{(\pi)^{1/2}S_w+1} \tag{19}$$

The freestream Knudsen number is expressed in terms of $d$ rather than $r$ since the former is more frequently used in the literature as the characteristic dimension. Retaining the variables on the righthand side of Eq. (19), a correlation parameter is defined as

$$\bar{K}_d = Kn_{\infty,d}/[(\pi)^{1/2}S_w+1] \tag{20}$$

### Correlation of Sphere Drag Data

Several sources[8-10,17-19] of low-density supersonic sphere drag data are presented in Fig. 2. The effect of $T_w$ and $S_\infty$ is accounted for by $C_D/C_{D_{fm}}$ and also by $\bar{K}_d$ through the wall speed ratio, $S_w$. A simple closed-form theoretical result[16] is also presented in Fig. 2.

Several other parameters have been used for correlating low-density drag data, most of which have been considered by Potter and Miller.[17] These parameters have also been considered for correlating the sphere drag data in Fig. 2; however, the correlation provided is not as good as that obtained by using $\bar{K}_d$.

### Correlation of High-Enthalpy Hypersonic Sphere Drag Data

Several sources[3,17,19-23] of hypersonic sphere drag data in terms of $Kn_{\infty,d}/[(\pi)^{1/2}S_w+1]$ are represented in Fig. 3 by the open symbols. Figure 3 reveals that the high-enthalpy, hypersonic, cold-wall data have larger values of $C_D/C_{D_{fm}}$ at the same values of $Kn_{\infty,d}/[(\pi)^{1/2}S_w+1]$ than the lower $M_\infty$ and/or lower enthalpy data considered in Fig. 2. A plausible explanation of this phenomenon might be that the energy of the molecules on the outermost surface of the sphere is rapidly increased above that corresponding to $T_w$ by collisions with the high energy freestream molecules, or the emitted molecules do not accommodate to the wall conditions, or a combination of both. If such is the case, an increase in drag occurs because of the higher momentum exerted on the body by the molecules which leave the surface, and also because emitted molecules will travel further before colliding with oncoming molecules and thereby produce less shielding according to the simple model of Ref. 7. The work of Kinslow and Potter[13] does not address this problem specifically; however, it does consider the correlation of surface pressures in low-density hypersonic flow, and, therefore, it is considered in search of an estimate of this effect.

Kinslow and Potter[13] used a two-sided Maxwellian velocity distribution function to describe the gas in the vicinity of a surface. For a bluff body stagnation region, their expression for the surface pressure, $p_w$, is approximated by

$$p_w/p_{i_{fm}} \approx (T_r/T_w)^{1/2} \tag{21}$$

where $p_{i_{fm}} = mn^- RT^-(T_w/T^-)^{1/2}$ is the pressure in a cavity in

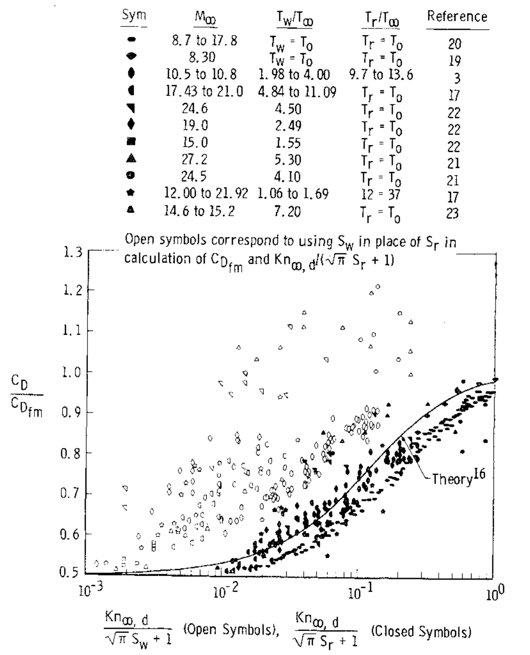


**Fig. 3 Correlation of high-enthalpy hypersonic sphere drag data.**

the surface where the cavity orifice is much smaller than the mean free path in the cavity, and the reference temperature $T_r$ is defined as

$$T_r = (T_o^- + T^+)/2 \tag{22}$$

The minus superscript denotes incoming molecules, and the plus superscript denotes outgoing molecules. The approximation made in Eq. (21) is the use of the exponent $\frac{1}{2}$ instead of the more accurate value which may vary between approximately $\frac{1}{2}$ and 1/(2.3). Since $p_{i_{fm}}$ is proportional to $(T_w)^{1/2}$, the actual surface pressure $p_w$ is given approximately by replacing $T_w$ with $T_r$ in $p_{i_{fm}}$.

It seems appropriate to investigate the change in the normalized drag data caused by using $T_r$ in place of $T_w$ in determining the pressure attributable to emitted molecules in calculating $C_{D_{fm}}$. The reference temperature, $T_r$, was used to calculate both $C_{D_{fm}}$ and $Kn_{\infty,d}/[(\pi)^{1/2}S_r+1]$; that is, $T_r$, was used in place of $T_w$ in calculating the wall speed ratio. These results are presented in Fig. 3 as solid symbols. For the data considered in Fig. 2, $T_r \approx T_w$, and therefore good correlation of the data in Figs. 2 and 3 is obtained by using $T_r$. Also, the scatter in the data of a given set of solid symbols is reduced somewhat; note particularly the $17.43 \leqq M_\infty \leqq 21.00$ data reported by Potter and Miller[17] in Fig. 3.

## IV. Conclusions

The investigation of the mean free path of molecules emitted from a surface, $\lambda_w$, has led to a relatively simple and accurate expression for $\lambda_w$. The parameter $\bar{K}_d$, which is proportional to $\lambda_w/d$, was shown to correlate experimental sphere drag data quite well for various flow and body surface variables.

It might be pointed out that the correlation parameter, $\phi$, introduced by Potter and Miller[17] is approximately the same as $(\bar{K}_d)^{-1}$ based on $S_w$ when $T_w/T_\infty = 1$. More precisely, $\phi^{-1} \approx Kn_{\infty,d}/[(\pi)^{1/2}S_w]$ which corresponds to the present zeroth-order solution, see Eq. (16). If $T_w/T_\infty = 1$, then $S_\infty/S_w = 1$ and since $S_\infty$ is relatively large, one has $\phi^{-1} \approx Kn_{\infty,d}/[(\pi)^{1/2}S_w+1] \approx Kn_{\infty,d}/[(\pi)^{1/2}S_w]$. For $T_w/T_o = 1$ and $\mu \sim T^{1/2}$, then again $\phi^{-1} \approx Kn_{\infty,d}/[(\pi)^{1/2}S_w]$; however, in this case $S_w \approx [\gamma/(\gamma-1)]^{1/2} \sim 0(1)$, and, therefore, more discrepancy exists between $\phi^{-1}$ and $\bar{K}_d$ because $S_w$ is not large. For $T_w/T_o = 1$ and $\mu \sim T$, there is significant discrepancy between $\phi^{-1}$ and $\bar{K}_d$. In this connection, it should be pointed out that Potter and Miller introduced $\phi$ as a high-speed flow parameter appropriate for real-gas nonequilibrium processes.

The problem of an appropriate reference temperature for molecules emitted from a surface has been discussed by other investigators.[5,6,11,12] Their approach to the problem was not the same as followed here. The only justification for the present use of the $T_r$, as derived by Kinslow and Potter,[13] is that it was shown to be useful in correlating surface pressures associated with the "orifice" effect for flow conditions similar to those where it has been shown herein to be useful in correlating force measurements. However, such justification is not sufficient, and a microscopic investigation of the problem seems to be in order.

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# Experimental Study of Supersonic Laminar Base Flow with and without Suction

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**Heat-transfer and pressure distributions in laminar separated flows downstream of rearward-facing steps with and without mass suction were investigated at Mach numbers around 4 for the conditions when the boundary-layer thickness was comparable to or larger than the step height. In both suction and no-suction cases, an increase of the step height resulted in a sharp drop of the base heating rates which then gradually recovered to less or near attached-flow values obtained with flat-plate configuration. Mass suction from the step base area increased the local heating rates, this effect was however relatively weak for laminar flows tested and the competing effect of the step height clearly predominated. It was found that even removal of the entire incoming boundary layer was not sufficient to raise the poststep heating rates above the flat-plate values. The base pressure in the no-suction, solid-step case correlated reasonably well with the step height-to-boundary-layer thickness ratio ($h/\delta$) and with the Reynolds number based on the step height ($Re_{\infty,h}$). Our experimental evidence indicated that entrainment conditions at separation may have a significant effect on the pressure distribution and flowfield behind the step. The results and trends observed in this study are discussed and explained qualitatively in terms of the simple flowfield models.**

## Nomenclature

$b$ = width of model
$h$ = step height
$H$ = enthalpy
$L$ = surface length preceding the step
$\dot{m}$ = mass flow rate
$M$ = Mach number
$P$ = pressure
$\dot{q}$ = heat flux
$Re_{\infty,h}$ = $U_\infty h/\nu_\infty$, Reynolds number based on $h$
$Re_{\infty,L}$ = $U_\infty L/\nu_\infty$, Reynolds number based on $L$
$T$ = absolute temperature
$U$ = velocity
$w$ = $\dot{m}_s/\rho_\infty U_\infty bh$, nondimensional mass suction-rate
$x$ = distance downstream of the leading edge
$\Delta x$ = distance downstream of the step
$\delta$ = boundary-layer thickness
$\delta^*$ = boundary-layer displacement thickness
$\rho$ = density
$\nu$ = kinematic viscosity

*Subscripts*

$b$ = base
$BL$ = boundary layer
max = maximum value
$ns$ = no suction
$o$ = stagnation conditions
ref = reference value
$s$ = suction; static
step = step location on the flat plate
$w$ = wall or surface value
$\infty$ = freestream conditions

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## Introduction

KNOWLEDGE of the flowfield past a rearward-facing step is important in the aerothermodynamic design of various flight configurations including hypersonic control surfaces and possible application of sliding metallic heat-shield panels for future space shuttle structures. One of the possible flow regimes associated with these configurations, and characteristic of the high altitude phase of re-entry, may involve a thick laminar boundary layer approaching the step (boundary-layer thickness $\delta$ comparable to or larger than the step height $h$) and possibility of mass suction from the base region. It is our intention to investigate heat-transfer and pressure distributions for such flow configurations.

The flowfield in the step region under no-suction conditions is determined by a strong interaction between the viscous recirculating region at the step base and the external inviscid flow. The studies of various aspects of a rearward-facing step or wedge are numerous; however, the vast majority of published works have centered on the case where the boundary-layer thickness is smaller than the step height, including investigations by Rom and Seginer,[1] Weiss and Weinbaum,[2,3] Scherberg and Smith,[4,5] Hama,[6] Batt and Kubota,[7] and more recent works by Erdos and Zakkay,[8] Wu and Su,[9] and Ohrenberger and Baum.[10]